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Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, May 2015
(2013 Scheme)
13.401 : ENGINEERING MATHEMATICS – III (E)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **(5×4=20)**

1. Prove that $f(z) = z^2$ is analytic.

2. Find the image of the circle $|z-1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$.

3. Evaluate $\int_C \frac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3} dz$ where C is $|z| = 1$.

4. Solve Max. $z = 6x_1 + x_2$ subject to $2x_1 + x_2 \geq 3$; $x_1 - x_2 \geq 0$; $x_1, x_2 \geq 0$.

5. Let W = span $\{x_1, x_2\}$ where $x_1 = \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix}$ and $x_2 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ construct an orthogonal bases $\{v_1\ v_2\}$ for W.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

6. a) Show that $u = \frac{1}{2} \log (x^2 + y^2)$ is harmonic and find its conjugate f (z).

   b) Show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$.

   c) Find the image of the circle $|z| = 2$ by the transformation $W = z + 3 + 2i$.

P.T.O.

7. a) In a two dimensional fluid flow the stream function is $\chi = \tan^{-1}\left(y/n\right)$. Find the velocity potential ϕ.

b) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ into points $w_1 = 1, w_2 = i, w_3 = -1$.

c) Show that $f(z) = e^{-x}(\cos y - i \sin y)$ is analytic.

Module – II

8. a) Evaluate $\int_C \frac{z+4}{z^2+2z+5}\,dz$ where C is the circle $|z+1-i| = 2$ using Cauchy's integral formula.

b) Obtain Taylor series to represent the function $\frac{z^2-1}{(z+2)(z+3)}$ in the region $|z| < 2$.

c) By using Cauchy Residue Theorem Evaluate $\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}\,dz$ where C is the circle $|z| = 3$.

9. a) Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2a\sin\theta+a^2}$, $0 < a < 1$.

b) Evaluate $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3}\,dz$ on the circle $|z| = 4$ and $|z| = 1$.

Module – III

10. a) Solve the LPP using Simplex Method

Minimize $z = x_2 - 3x_3 + 2x_5$

Subject to

$3x_2 - x_3 + 2x_5 \leq 7$;

$-2x_2 + 4x_3 \leq 12$;

$-4x_2 + 3x_3 + 8x_5 \leq 10$;

$x_2, x_3, x_5 \geq 0$

b) Use Big-M-method to solve

Maximize $z = 2x_1 + 3x_2$

Subject to

$x_1 + 2x_2 \leq 4$

$x_1 + x_2 = 3$

$x_1, x_2 \geq 0$

11. a) Solve the LPP

Max. $z = 5x_1 + 3x_2$

Subject to

$x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0$

b) Solve by Big-M method

Max. $z = 3x_1 + 2x_2$

Subject to

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

Module – IV

12. a) Find the basis for row space, column space and null space given

$$A = \begin{pmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{pmatrix}$$

b) Show that $\{u_1, u_2, u_3\}$ is an orthogonal set for

$$u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} -1/2 \\ 2 \\ 7/2 \end{pmatrix}$$

13. a) Let $A = \begin{pmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{pmatrix}$

i) If the column space of A is a subspace of R^K. Determine K.

ii) If the null space of A is the subspace of R^K. Find K.

b) Let $X_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} X_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} X_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ construct an orthogonal basis for W if

$\{X_1, X_2, X_3)$ is linearly independent and basis for the subspace of R^4.
